Uncertainty-Based Optimization of Site Characterization Using CPT

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ABSTRACT: In the light of complexities and difficulties involved in geotechnical site characterization, an uncertainty-based methodology based on a large number of CPT records for optimizing CPT site characterization is proposed. In this paper, the main features of the Uncertainty-Based Optimization of CPT site characterization are discussed and an example for planning an optimum site characterization programme for foundation design is demonstrated.

1 INTRODUCTION

One of the major difficulties involved in CPT site characterization is associated with the determination of the appropriate amount and extent of works such as the layer depth, the vertical testing interval and the spacing of soundings. Traditional procedures for CPT site characterization vary with the geological conditions and general procedures of the company involved. Borings or soundings are generally taken at locations of heavily loaded structures and location of difficult soil conditions. The amount and extent of works are generally selected based on engineering experience and the benefits obtained from the characterization are usually not quantified (Hvorslev 1949, Weltman & Head 1983). Therefore, the traditional site characterization program is frequently ineffective. More specifically, there are difficulties in the interpretation of observations and measurements derived from site characterization. These shortcomings may be attributed due to the lack of mechanisms for proper accounting for the uncertainties involved in the site characterization process. Numerous researchers have studied the effects of uncertainties involved in the geotechnical analysis and design. (Vanmarcke 1983; Jaksa et al. 2004; Phoon & Kulhawy 1999; Fenton & Griffiths 2003; Sivakumar Babu & Mukesh 2004; Uzielli et al. 2005; & Wei et al. 2005 and it has been pointed out that the natural soil variability predominates among other sources of uncertainties. Therefore, in order to obtain a more effective geotechnical site characterization, an uncertainty-based approach with emphasis on the quantificationn of soil variability appears to be warranted. By modeling the engineering properties of soils as a random field, the soil variability can be quantified in a probabilistic framework. The variability of soil properties is expressed by the spatial coefficient of variation and the scale of fluctuation. In this study a method has been developed for determining the scale of fluctuation for different types of soils using a large number of CPT data carried out in Macau. This parameter is used for determining the vertical testing interval and exploration spacing for CPT soundings. Furthermore, a method has also been developed for estimating the number of tests required for optimizing CPT site characterization.

2 PROBABILISTIC MODELING OF SOIL PROFILE

In order to have reliable solution to the geotechnical problems, probabilistic modeling of soil profile should be considered for analysis. In a probabilistic soil profile, at least one of the profile characteristic is treated as a random function of one or more of the coordinates. Recent approaches based on a random field model (Baecher 1984, Tang 1984, Phoon & Kulhawy 1999, Fenton & Griffiths 2003, Jaksa et al. 2004) proposed by (Vanmarke 1977) provide statistical procedures for capturing the variable nature and interdependence of soil properties. More specifically, these models facilitate a proper accounting for the reduction in uncertainty in test results. In this investigation, the soil profiles are modelled by a random field; the cone tip resistances along each soil profile of different types of soils are treated as separate random variables. In other to have an adequate description of the spatial variability of the soil profiles, two parameters, namely, the scale of fluctuation, δ, and the coefficient of variation, V, based on cone tip resistances have been evaluated and are termed as variability parameters.

2.1 *Evaluation of scale of fluctuation*

The similarity in value for soil property at closely neighbouring locations can be described by the scale of fluctuation, δ. The scale of fluctuation gives an indication of the degree of variability of a soil profile. Figure 1 illustrates the meaning of this parameter.

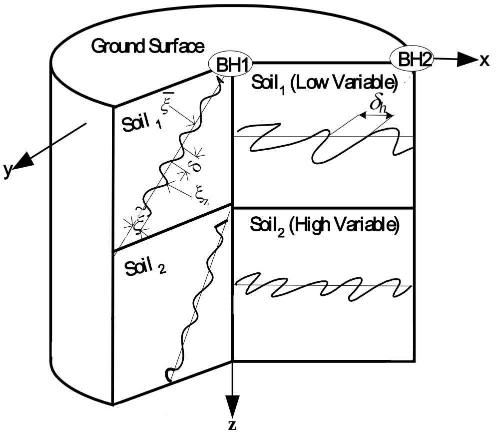

Figure1. Scale of fluctuation of a soil property

where δ represents the scale of fluctuation varied with depth and δ_h represents the horizontal scale of fluctuation. It shows that the soil property ξ_z fluctuates about its mean value $\bar{\xi}$ with its standard deviation $\tilde{\xi}$ of the entire layer. A highly variable profile will have a smaller δ while a slowly varying profile will result in larger δ. The scale of fluctuation of any stratum is referred to as the distance within which the soil property shows relatively strong correlation. In view of this fact, δ is termed as correlation distance in this study and is derived from the variance function Γ^2 (Vanmarke 1977). Therefore the variance function of cone tip resistances is predicted from:

$$\Gamma^2(Z_n) = \frac{\tilde{q}_{cn}^{\ 2}}{\tilde{q}_c^{\ 2}} \qquad (1)$$

Where $\tilde{q}_{cn}^{\ 2}$ is the variance of the derived moving average series of degree n, and $\tilde{q}_c^{\ 2}$ is the variance of the original data. If the spacing of the data is Δz, for large values of n, these predicted values had approached the theoretical values:

$$\Gamma_n^2 n \Delta z \cong \delta \qquad (2)$$

Equation 2 was solved graphically by plotting the Γ_n versus n (Vanmarke 1983). However, it involves trial and error procedures, hence considerable uncertainty will be involved. In view of this fact, a more reasonable method is proposed in this investigation for better estimation of δ. Equation 2 can be written as:

$$\gamma_n = \delta - \Gamma_n^2 n \Delta z \qquad (3)$$

Where γ_n is the residual value and Γ_n^2 is the experimental values and δ is a deterministic constant. Therefore, δ can be estimated by minimizing the sum of the squared errors and finally leads to:

$$\delta = \sum_{n=1}^{M} \left(\frac{\Gamma_n^2 n \Delta z}{M} \right) \qquad (4)$$

Based on equation 4, the correlation distance can be evaluated in a more systematic way especially when the amount of data is large and therefore considerable uncertainties can be minimized. Furthermore, the coefficient of variation, V, which is another soil variability parameter is a dimensionless parameter expressed by the ratio of the standard deviation to the mean. It provides a more stable measure of consistency than its constituents represent the variability of soil properties. Once the soil variability parameters for each soil type are estimated, the data can provide a potential benefit in the design of site characterization programme and in the evaluation of their effectiveness. Table 1 presents the statistical evaluation of δ, its uncertainty level $V(\delta)$ and the coefficient of variation of cone tip resistances $V(q_c)$ obtained from the proposed and Vanmarke's methods.

Table 1. Statistical evaluation of δ and V for $q_{c.}$

Soil Types	Methods	δ(m)			
		n*	Range	*Mean*	$V(\delta)$
Clay	- Vanmarke's	42	0.34~0.48	0.46	0.35
$V(q_c)$ = 0.12	- Proposed	42	0.46~0.50	0.48	0.11
Sitly Clay	- Vanmarke's	38	0.32~0.49	0.43	0.47
$V(q_c)$ =0.28	- Proposed	38	0.42~0.45	0.44	0.15

* n means the number of cases

It is observed that the variability of δ obtained from the proposed method is relatively low and its values generally lie in a narrow range whereas the level of uncertainty of δ obtained from Vanmarke's approach is especially high and the mean values lie in a wider range compared to the proposed approach for both types of soils. Similarly, the horizontal correlation distance δ_h has also been estimated from the same CPT database derived by placing a set of CPT soundings at constant spacing perpendicular to the vertical sampling direction as shown in Figure 2.

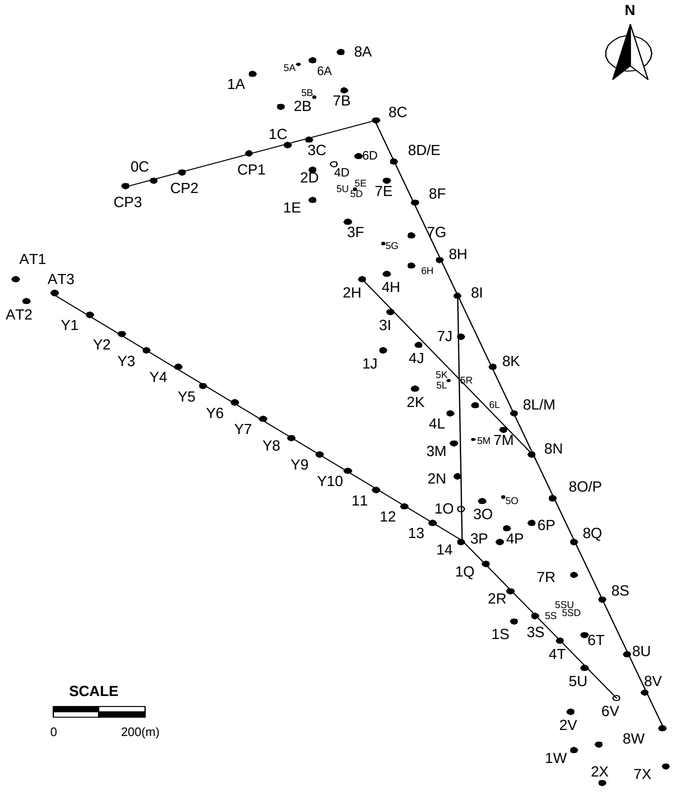

Figure 2. Typical planning for horizontal analysis.

Estimation is proceeded to successive elevations separated by a predetermined depth layer by layer within the same profile and each soil profile is analyzed in the same way. The results of the horizontal analysis are shown in Table 2.

Table 2. Variability Parameters-Horizontal Analysis

Soil Types	Clay		Silty clay	
Depth (m)	δ_h (m)	$V_h(q_c)$	δ_h (m)	$V_h(q_c)$
9-12	285	0.12	245	0.19
12-15	286	0.12	224	0.18
15-18	282	0.13	212	0.20
18-21	279	0.11	218	0.22
Average	283	0.12	225	0.20

It is observed that the values of $V_h(q_c)$ in each depth level are very close to each other although there is a gentle decrease in δ_h with depth. It shows that the variability of both types of soils in the horizontal direction is very small with depth.

3 OPTIMUM PLANNING FOR CPT SITE CHARACTERIZATION

3.1 *Optimum vertical testing interval and exploration spacing*

A fundamental requirement of sampling, in the statistical sense, is random selection of the samples. Such that individual element of the samples will be independent of each other. Otherwise the information obtained will be somewhat redundant. It is obvious that the correlation distance can give an indication of choosing how far the individual elements of the sample can be regarded as independent from each other. It is proposed that in order to avoid correlation among the properties of different samples, sampling distances should be chosen which are large in comparison with the correlation distance in order to have an optimum planning of the extent for CPT characterization.

3.2 *Optimum number of CPT soundings*

For test results obtained from a set of samples to be meaningful, two main conditions should be satisfied. First, the samples must be representative of the material from which they are taken. Second, the number of samples must be sufficient. These circumstances refer to the fact that correct sampling and testing methodology can be elaborated only by statistical methods, adjusted to the given geotechnical conditions. Once the properties are normalised, if it is warranted, then the second moment statistics are evaluated. For all possible values of a property within a layer, there is a true mean, μ, and variance, σ^2. However, in many cases the form of density function of a soil property, ξ, may not be known. Therefore, the value of the mean of a population of a soil property has to be estimated from a set of sample values and the estimate of the mean, $\bar{\xi}$, and variance, $\tilde{\xi}^2$, of CPT data are used for analysis in this study.

In order to determine the number of tests required for a proposed project, it is suggested to estimate how far away the sample value is from the actual mean value of the population. This is achieved by a term called the standard deviation of the mean $D(\bar{\xi})$ and is computed from:

$$D(\bar{\xi}) = \frac{\tilde{\xi}}{N}\sqrt{\frac{M-N}{M-1}} \tag{5}$$

Where $\tilde{\xi}$ is the standard deviation of the population, *N* is the required number of test and *M* is the total number of elements of the sample. When $\tilde{\xi}$ is not known, as is generally the case, $\tilde{\xi}$ has been approximated by the empirical standard deviation, *S* and as according to a variant of the central limit distribution theorem, the mean soil property, $\bar{\xi}$, was approximately of a normal distribution if *N* is sufficiently large, the interval

$$\bar{\xi} \pm \lambda \frac{S}{\sqrt{N}}\sqrt{\frac{M-N}{M-1}} \tag{6}$$

covers approximately the expected value of the population consisting of *M* elements, where λ is a constant dependent on the level of confidence. In practice *M* can be regarded as infinite relative to *N*, while at the same time n is not sufficiently large for permitting to regard $\bar{\xi}$ as normally distributed. So, Equation 6 must be modified as follows:

$$\bar{\xi} \pm t_\alpha \frac{S}{N} \tag{7}$$

Where t_α is the function value of the Student's t-distribution belonging to the chosen probability level α and degrees of freedom *N-1*. As the number of degrees of freedom (n-1) is increased, the Student's t-distribution approached a normal distribution. If the expected value of the population is μ, then the condition that the difference $\bar{\xi} - \mu$ should be smaller with a probability of α than the value Δ according to the Equation 7 is:

$$\frac{t_\alpha}{\sqrt{N}} < \frac{\Delta}{S} \tag{8}$$

Dividing the numerator and the denominator of the fraction standing on the right side of Equation 8 by $\bar{\xi}$, the following expression is obtained:

$$\frac{t_\alpha}{\sqrt{N}} < \frac{\Delta_r}{V} \tag{9}$$

Where Δ_r is the maximum error committed in the estimation of the mean and V is the coefficient of variation. It is assumed that any layer is fully characterized when the average value obtained from the data for that layer is within Δ_r% of the actual average which is unknown. The required number of CPT values, n, needed to estimate the mean to the above precision, with a confidence level, can be estimated by using Equation 9. Once the required number of CPT data is determined, the maximum number of tests can be performed in each sounding is estimated by dividing the thickness of soil layer by the testing interval and the required number of CPT soundings, B_N, can be estimated by dividing the total number of the tests, N, by the maximum number of the tests, T_N, at each sounding.

3.3 Numerical example and discussions

To illustrate potential applications of the proposed approach for CPT characterization planning, a project involving the design of foundation is considered. Based on the prior information of CPT data obtained near the site of the proposed project, the layer thickness for each type of soils is estimated and the variability parameters are computed. Tables 3 summarized the results obtained from the proposed method.

Table 3. Results based on Tip Resistances

	Soil Types	Thickness(m)	V(%)	δ_v (m)	δ_h(m)
Input	Silty Clay	20	0.19	0.44	225
	Clay	8	0.17	0.48	283

	Soil Types	Δr (%)	$\Delta r / V$	α (%)	N	T_N	B_N
Results	Silty Clay	5	0.2632	90	29	29	1
				95	47	47	1
				99	94	66	2
		10	0.5263	90	8	8	1
				95	13	13	1
				99	26	26	1
	Clay	5	0.2941	90	20	20	1
				95	33	33	1
				99	66	39	2
		10	0.5882	90	6	6	1
				95	10	10	1
				99	19	19	1

It can be seen from Table 3, for instance, for maximum error of 10% and a confidence level of 95%, the number of CPT data (tip resistance) required for the silty clay of higher variability (e.g., $V = 0.19$) is 13 while for soil with lower variability such as clay with V equal to 0.17, the required number of CPT data is reduced to 10. However, if a higher confidence level of 99% is required in the more variable soil, number of data should be increased to 26. Furthermore, the number of CPT soundings also affected by the layer thickness, Δ_r and α. With regard to the exploration spacing, the horizontal correlation distances is chosen by selecting the largest value of δ_h among all the soil types in order to avoid redundant information of sampling. As a result, the spacing of 300 m is appropriate for the proposed project.

CONCLUSIONS

In view of the difficulties and complexities involved in geotechnical site characterization, an uncertainty-based methodology for designing an optimum CPT site characterization programme have been developed in this study. It provides a useful guideline for designing a more economical and reliable foundation. From this study, the following conclusions have been drawn:

(1) A method has been developed for determining the vertical and horizontal correlation distances for planning CPT site characterization which are used for determining the vertical testing interval and the exploration spacing of CPT soundings.

(2) Based on the correlation distances and the coefficient of variation, a method is proposed for obtaining the amount of CPT tests required for a proposed project.

REFERENCES

Babu, G. L.S. & Mukesh, M.D. 2004. Effect of soil variability on reliability of soil slopes. Geotechnique 54(5):335–337.

Baecher, G.B. 1984. Need for probablistic characterization. Proc. of Probabilistic characterization of soil properties: Bridge between theory and practice, Atlanta, ASCE. New York:1-1

Fenton, G.A.&Griffiths, D.V. 2003. Bearing capacity prediction of spatially random c-ϕ soils. Can Geotech J 40:54–65

Hvorslev, M. J.1949. Subsurface Exploration and Sampling of Soils for Civil Engineering Purpose, Waterways Experiment Station, Vicksburg.

Jaksa, M.B., Yeong, K.S., Wong, K.T.&Lee, S.L. 2004. Horizontal spatial variability of elastic modulus in sand from the dilatometer. In: Proc. 9th Australia New Zealand conference on geomechanics, Auckland, pp 289–294

Phoon, K.K.& Kulhawy, F.H. 1999. Evaluation of geotechnical property variability. Can Geotech J 36(4):625–639

Tang, W. H. 1984. Principles of probablistic characterization of soil properties. Proc. Probablistic characterization of soil properties: Bridge between theory and practice, Atlanta, ASCE. New York:74-89

Vanmarke, E.H. 1977. Probabilistic modeling of Soil Profiles, Journal of the Geotechnical Engineering Division, ASCE, Vol. 103, No. GT11, 1227-1246.

Vanmarcke, E.H. 1983. Random fields: analysis and synthesis. MIT Press, Cambridge.

Uzielli, M.,Vannucchi, G.&Phoon, K.K. 2005. Normal field characterization of stress-normalised cone penetration testing parameters. Geotechnique 55(1):3–20

Wei, L., Tumay, M.T. &Abu-Farsakh, M.Y. 2005. Field testing of inclined cone penetration. Geotech Test J ASTM 28(1):31–41.

Weltman, A. J. & Head, J. M. 1983. Site Investigation Manual, CIRIA Special Publication 25, Construction Industry Research and Information Association, London, 144 p.